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# W11.5

# **MEMORANDUM**

Date: 16 January 2003

To: Commissioners and Interested Parties

From: Mark Johnsson, Staff Geologist

Subject: Establishing development setbacks from coastal bluffs

# **STAFF NOTE**

Consistency with section 30253 of the Coastal Act requires that:

New development shall:

- (1) Minimize risks to life and property in areas of high geologic, flood, and fire hazard.
- (2) Assure stability and structural integrity, and neither create nor contribute significantly to erosion, geologic instability, or destruction of the site or surrounding area or in any way require the construction of protective devices that would substantially alter natural landforms along bluffs and cliffs.

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This section requires that new development be located such that it will not be subject to erosion or stability hazard over the course of its design life. Further, the last clause requires the finding that no seawall, revetment, jetty, groin, retaining wall, or other shoreline protective structure, inasmuch as such a structure would substantially alter natural landforms along bluffs and cliffs, will be needed to protect the development over the course of its design life. The Commission has found on many occasions that siting new development away from eroding bluffs is the preferred means of assuring consistency with this section, and the establishment of bluff-top setbacks for new development is an integral part of most local coastal programs. Further, the State's draft Policy on Coastal Erosion Planning and Response states that avoidance of geologic hazards, such as eroding coastal bluffs, should be the primary means of safeguarding new development. Accordingly, the determination of what constitutes an adequate setback is a critical component of the analysis of proposals for new development.

Because coastal bluffs are dynamic, evolving landforms, establishing appropriate development setbacks from coastal bluffs is far more challenging than it is for manufactured or natural slopes not subject to erosion at the base of the slope. The mechanisms of coastal bluff retreat are complex, but can be grouped into two broad categories. Bluff retreat may occur suddenly and catastrophically through slope failure involving the entire bluff, or more gradually through grain-by-grain erosion by marine, subaerial, and ground water processes. For both processes, the setback must be adequate to assure safety over the design life of the development.

In an effort to clarify the analytical procedures undertaken by Coastal Commission staff in evaluating proposed development setbacks, the Commission's staff geologist made two presentations at the *California and the World Ocean '02* conference held in Santa Barbara in October 2002. These presentations were combined into a single manuscript to be published in the proceedings volume for that Conference, which is attached to this staff report.

In order to bring these procedures before the Commission, and to further the exposure of them to the public, the staff geologist will brief the Commission on this methodology at the February 2003 hearing. This methodology does not represent a formal policy or position of the Coastal Commission. In fact, there may be other appropriate methodologies to establish development setbacks, and the Commission has the discretion to base a decision on any method that it finds technically and legally valid. Further, as new techniques and information become available, these methodologies may change. Nevertheless, the type of analysis outlined here represents the current analytical process carried out by Coastal Commission staff in evaluating proposals for new development on the California coast, and in recommending action upon those proposals to the Commission. The Commission then makes its decisions on a case-by-case basis, based upon the site-specific evidence related to the particular development proposal.

Attachment: Preprint of manuscript entitled "Establishing development setbacks from coastal bluffs," by Mark J. Johnsson, to appear in *Proceedings, California and the World Ocean, '02*, Orville Magoon, ed., 21 p.

#### **Establishing Development Setbacks from Coastal Bluffs** Mark J. Johnsson<sup>1</sup>

#### Walk J. John

# Abstract

Responsible development, and California law, requires that coastal development be sited a sufficient distance landward of coastal bluffs that it will neither be endangered by erosion nor lead to the construction of protective coastal armoring. In order to assure that this is the case, a development setback line must be established that places the proposed structures a sufficient distance from unstable or marginally stable bluffs to assure their safety, and that takes into account bluff retreat over the life of the structures, thus assuring the stability of the structures over their design life. The goal is to assure that by the time the bluff retreats sufficiently to threaten the development, the structures themselves are obsolete. Replacement development can then be appropriately sited behind a new setback line. Uncertainty in the analysis should be considered, as should potential changes in the rate of bluff retreat and in slope stability. The deterministic approach presented here is based on established geologic and engineering principals, and similar approaches have been used to establish development setbacks from slope edges throughout the world for some time. Alternative approaches based on probabilistic methods may allow, however, for better quantification of uncertainties in the analysis. Although probabilistic coastal hazard assessment is in its infancy and data needs are large, the approach shows great promise. Developing probabilistic methods for establishing development setbacks should be a goal for future coastal zone management in California.

# Introduction

In an era of sea-level rise such as has persisted on Earth for the past ~20,000 years (Curray 1965; Emery and Garrison 1967; Milliman and Emery 1968), the landward recession of coastal bluffs is an inevitable natural process wherever tectonic or isostatic uplift rates are lower than the rate of sea-level rise. New structures should be sited a sufficient distance landward of coastal bluffs that they will neither be endangered by erosion nor require the construction of coastal armoring to protect them from erosion over their design life. Because coastal bluffs are dynamic, evolving landforms, establishing responsible development setbacks from coastal bluffs is far more challenging than it is for manufactured or natural slopes not subject to erosion at the base of slope. Although internationally agreed-upon methods for establishing setbacks from static slopes have been developed, and codified in the International Building Code, no such consensus has emerged with respect to setbacks from dynamic slopes such as coastal bluffs. This paper presents a methodology for establishing such setbacks given the types of data generally available through relatively in expensive geologic studies.

Relatively little work has been undertaken towards developing rational methodologies for establishing development setbacks from bluffs and cliffs. Coastal development setbacks have generally focused primarily on beach erosion, rather than on coastal bluff recession (e.g., Healy 2002). Generally, the approach has been to simply

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extrapolate historic long-term erosion rates into the future, and establish setbacks at a particular predicted future shoreline position. This approach does not work well for shorelines with coastal bluffs, where the setback also must consider the possibility of bluff collapse (see Priest 1999 for a discussion of these issues). Komar and others (2002) presented a methodology for establishing setbacks for use on coasts where the principal hazards are wave runup and storm surge. They showed how their method could be extended to use on coasts with sea cliffs by determining the average number of hours that a sea cliff would be subject to wave attack. Their method does not, however, include a quantitative assessment of bluff stability. Given the significance of the coastal erosion threat in California, where public safety, financial investments, and environmental resources are at stake, and given the call for action urged by such recent national studies as the Heinz Center's FEMA-sponsored studies (The Heinz Center 2000a; 2000b), it is critical that a rational method be established for establishing development setbacks on coastal bluff tops.

The California Coastal Act (California Public Resource Code Sections 30000 *et seq.*) regulates coastal development in California. Section 30253 states, in part, that:

New development shall:

. . .

- (1) Minimize risks to life and property in areas of high geologic, flood, and fire hazard.
- (2) Assure stability and structural integrity, and neither create nor contribute significantly to erosion, geologic instability, or destruction of the site or surrounding area or in any way require the construction of protective devices that would substantially alter natural landforms along bluffs and cliffs.

This law requires that new development be sited in such a way that it will not be subject to erosion or stability hazard over the course of its design life. Further, the last clause requires the finding that no seawall, revetment, jetty, groin, retaining wall, or other shoreline protective structure will be needed to protect the development over the course of its design life.

The principal challenge in meeting these requirements is predicting the amount and timing of coastal erosion to be expected at a particular site. The landward retreat of coastal bluffs is far from uniform in space or time (Komar 2000). Marine erosion tends to be concentrated at points and headlands due to wave refraction, occurs more quickly in weak rocks, and may vary along a coastline as these and other factors vary (Honeycutt et al. 2002). Further, coastal bluff retreat tends to be temporally episodic due to a variety of external and internal factors.

The mechanisms of coastal bluff retreat are complex (Emery and Kuhn 1982; Sunamura 1983; Vallejo 2002), but can be grouped into two broad categories. Bluff retreat may occur suddenly and catastrophically through slope failure involving the entire bluff, or more gradually through grain-by-grain erosion by marine, subaerial, and ground water processes. The distinction between the two categories may be blurred in some cases—"grains" may consist of relatively large blocks of rock or shallow slumps, for example. Nevertheless, in establishing structural setbacks it is important to evaluate the susceptibility of the bluff to both catastrophic collapse and to more gradual erosion and retreat.

For both slope stability and long-term bluff retreat by "grain-by-grain" erosion, the setback must be adequate to assure safety over the design life of the development. For this reason, it is necessary to specify the design life of the structure. Many Local Coastal Programs (the implementation of the California Coastal Act at the local government level) specify a particular value, although the Coastal Act itself does not. The most commonly assumed design lives for new development range from 50 to 100 years; the most common value is 75 years. The reasoning behind establishing a setback based on the design life is that by the time the bluff retreats sufficiently to threaten the structure, the structure is obsolete and is ready to be demolished for reasons other than encroaching erosion. Replacement development can then be appropriately sited at a new setback, appropriate for conditions at the time of its construction. This process may be thwarted by limitations imposed by parcel size, and Constitutional takings issues may complicate land use decisions. Nevertheless, the only alternative to an armored coast-with all of its attendant impacts-is to continually site, and reposition, development in harmony with coastal erosion as it inevitably moves the shoreline landward.

What follows is the methodology employed by the staff of the California Coastal Commission in evaluating setbacks for bluff top development. I would suggest that this methodology is useful on other coasts with coastal bluffs, as well. This methodology does not represent a formal policy or position of the Coastal Commission. In fact, there may be other appropriate methodologies to establish development setbacks, and the Commission has the discretion to base a decision on any method that it finds technically and legally valid. Any such alternative methods should, however, be at least as protective of coastal zone resources as those outlined here. Further, as new techniques and information become available, these methodologies may change. Nevertheless, the type of analysis outlined here represents the current analytical process carried out by Coastal Commission staff in evaluating proposals for new development on the California coast, and in recommending action upon those proposals to the Commission. The Commission then makes its decisions on a case-by-case basis, based upon the site-specific evidence related to the particular development proposal.

# **Definition of "Bluff Edge"**

Development setbacks normally are measured from the upper edge of the bluff top. Accordingly, a great deal of effort often is focused on defining that "bluff edge." The bluff edge is simply the line of intersection between the steeply sloping bluff face and the flat or more gently sloping bluff top. Defining this line can be complicated, however, by the presence of irregularities in the bluff edge, a rounded or stepped bluff edge, a sloping bluff top, or previous grading or development near the bluff edge. Accordingly, a set of standards for defining the bluff edge is necessary.

Under the California Coastal Act, the bluff edge is defined as:

... the upper termination of a bluff, cliff, or seacliff. In cases where the top edge of the cliff is rounded away from the face of the cliff as a result of erosional processes related to the presence of the steep cliff face, the bluff line or edge shall be defined as that point nearest the cliff beyond which the downward gradient of the surface increases more or less continuously until it reaches the general gradient of the cliff. In a case where there is a steplike feature at the top of the cliff face, the landward edge of the topmost riser shall be taken to be the cliff edge..." (California Code of Regulations, Title 14, §13577 (h) (2).

This definition is largely qualitative, and the interpretation of the topographic profile to yield a bluff edge determination at any given coastal bluff may be subject to various interpretations. Accordingly, it may be useful to use more quantitative means to define "bluff edge." One approach, adopted, for example, by the City of Laguna Beach, is to define the bluff edge as that point at which the coastal bluff attains a certain specified steepness. This steepness is equivalent to the first derivative of the topographic profile. Such a definition may, however, be inconsistent with the legal definition above. Further, ambiguous results may be obtained when the upper portion of the bluff fluctuates around the specified steepness value. Better results may be obtained by finding the point at which the second derivative, the rate of change in steepness, of the topographic profile increases sharply. This approach may be amenable to computer analysis, although such analysis is rarely employed.

The position of the bluff edge may be changed by a variety of processes, natural and anthropogenic. Most obvious is the landward retreat of the bluff edge through coastal erosion. A bluff edge also may move seaward, through tectonic processes, but such movement is rare and usually small on human time scales. More significant is the anthropogenic modification of the bluff edge by grading or the construction of structures. A landward shift of the bluff edge commonly occurs through cutting into and removing natural materials during grading operations or the construction of seawalls. Conversely, placing artificial fill on or near the bluff edge generally does not alter the position of the natural bluff edge; the natural bluff edge still exists, buried beneath fill, and the natural bluff edge is used for purposes of defining development setbacks.

# **Slope Stability**

Once the bluff edge is located, the first aspect to consider in establishing development setbacks from the bluff edge is to determine whether the existing coastal bluff meets minimum requirements for slope stability. If the answer to this question is "yes," then no setback is necessary for slope stability considerations. If the answer is "no," then the distance from the bluff edge to a position where sufficient stability exists to assure safety must be found. In other words, we must determine how far back from the unstable or marginally slope must development be sited to assure its safety. We are guided in this analysis by the industry-accepted standards for artificial slopes (codified in many local grading ordinances), which require that a particular minimum "factor of safety" against landsliding be attained. A more difficult situation is the case of overhanging or notched coastal bluffs, or bluffs undermined by sea caves.

*Landslides.* Assessing the stability of slopes against landsliding is undertaken through a quantitative slope stability analysis. In such an analysis, the forces resisting a potential landslide are first determined. These are essentially the strength of the rocks or soils making up the bluff. Next, the forces driving a potential landslide are determined. These forces are the weight of the rocks as projected along a potential slide surface. The resisting forces are divided by the driving forces to determine the "factor of safety." A value below 1.0 is theoretically impossible, as the slope would have failed already. A value of 1.0 indicates that failure is imminent. Factors of safety at increasing values above 1.0 lend increasing confidence in the stability of the slope. The industry-standard for new development is a factor of safety of 1.5, and many b-cal grading ordinances in California and elsewhere (including the County of Los Angeles, and the Cities of Irvine, Malibu, and Saratoga, among others) require that artificial slopes meet this factor of safety.

A slope stability analysis is performed by testing hundreds of potential sliding surfaces. The surface with the minimum factor of safety will be the one on which failure is most likely to occur. Generally, as one moves back from the top edge of a slope, the factor of safety against landsliding increases. Therefore, to establish a safe setback for slope stability from the edge of a coastal bluff, one needs to find the distance from the bluff edge at which the factor of safety is equal to 1.5.

Inherent in the calculation of a slope stability analysis is the shape (topographic profile) and geologic makeup of the coastal bluff. There are many ways to calculate the forces involved in slope stability analyses. All methods must consider such factors as rock or soil strength, variations in rock and soil strength values due to different types of materials making up the slope, anisotropy in these values, and any weak planes or surfaces that may exist in the slope (Abramson et al. 1995). More subtly, other factors that must be considered include: pore water pressure, which produces a buoyant force that reduces the resisting forces, the particular failure mechanism that is most likely (e.g., a block slide mechanism vs a circular failure mechanism), and seismic forces. Seismic forces normally are considered through a separate analysis, in which a force equal to 15% of the force of gravity is added to the driving forces. Because seismic driving forces are of short duration, a factor of safety of 1.1 generally is considered adequate to assure stability during an earthquake. This type of analysis is fairly crude, and other methods for evaluating slope stability based on maximum permanent displacement experienced during earthquakes do exist, but the pseudostatic method represents the current standard of practice for most development in California (Geotechnical Group of the Los Angeles Section of the American Society of Civil Engineers 2002). Guidelines for conducting slope stability analyses for review by the California Coastal Commission are presented in Table 1.

# Table 1. Guidelines for performing quantitative slope stability analyses

- 1) The analyses should demonstrate a factor of safety greater than or equal to 1.5 for the static condition and greater than or equal to 1.1 for the seismic condition. Seismic analyses may be performed by the pseudostatic method or by displacement methods, but in any case should demonstrate a permanent displacement of less than 50 mm.
- 2) Slope stability analyses should be undertaken through cross-sections modeling worst case geologic and slope gradient conditions. Analyses should include postulated failure surfaces such that both the overall stability of the slope and the stability of the surficial units is examined.
- 3) The effects of earthquakes on slope stability (seismic stability) may be addressed through pseudostatic slope analyses assuming a horizontal seismic coefficient of 0.15g. Alternative (displacement) methods may be useful, but should be in conformance with the guidelines published by the Geotechnical Group, American Society of Civil Engineers, Los Angeles Section (2002).
- 4) All slope analyses should ideally be performed using shear strength parameters (friction angle and cohesion), and unit weights determined from relatively undisturbed samples collected at the site. The choice of shear strength parameters should be supported by direct shear tests, triaxial shear test, or literature references, and should be in conformance with the guidelines published by the Geotechnical Group, American Society of Civil Engineers, Los Angeles Section (2002).
- 5) All slope stability analyses should be undertaken with water table or potentiometric surfaces for the highest potential ground water conditions.
- 6) If anisotropic conditions are assumed for any geologic unit, strike and dip of weakness planes should be provided, and shear strength parameters for each orientation should be supported by reference to pertinent direct sheer tests, triaxial shear test, or literature references.
- 7) When planes of weakness are oriented normal to the slope or dip into the slope, or when the strength of materials is considered homogenous, circular failure surfaces should be sought through a search routine to analyze the factor of safety along postulated critical failure surfaces. In general, methods that satisfy both force and moment equilibrium, such as Spencer's (Spencer 1967; 1973), Morgenstern-Price (Morgenstern and Price 1965), and General Limit Equilibrium (Fredlund et al. 1981; Chugh 1986) are preferred. Methods based on moment equilibrium alone, such as Simplified Bishop's Method (Bishop 1955) also are acceptable. In general, methods that solve only for force equilibrium, such as Janbu's method (Janbu 1973) are discouraged due to their sensitivity to the ratio of normal to shear forces between slices (Abramson et al. 1995).
- 8) If anisotropic conditions are assumed for units containing critical failure surfaces determined above, and when planes of weakness are inclined at angles ranging from nearly parallel to the slope to dipping out of slope, factors of safety for translational failure surfaces should also be calculated. The use of a block failure model should be supported by geologic evidence for anisotropy in rock or soil strength. Shear strength parameters for such weak surfaces should be supported through direct shear tests, triaxial shear test, or literature references.

**Establishing a safe setback line.** Once the stability of the coastal bluff has been assessed, the development setback line to assure safety from marginally stable slopes is simply the line corresponding to a factor of safety of 1.5 (static) or 1.1 (pseudostatic), whichever is further landward. In establishing this line one can either use a single cross section and specify a single distance from the bluff edge at which the factor of safety rises to 1.5 (or 1.1 for the pseudostatic case), or use several cross sections and contour the factors of safety on the bluff top. Then, by choosing the 1.5 contour (or 1.1 for the pseudostatic case, if it lies further landward), a setback line is established. The latter method generally is necessary for large or complicated sites.

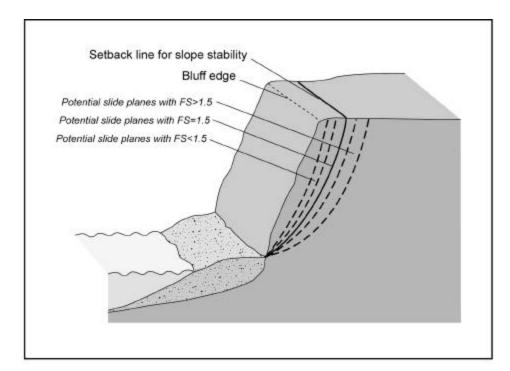


Figure 1. Establishing a development setback for slope stability. The potential slide plane possessing a defined minimum standard of stability is identified, and its intersection with the bluff edge is taken as a minimum development setback. The minimum standard for stability is usually defined as a factor of safety (FS) against sliding of 1.5 for the static case, or 1.1 for a pseudostatic (seismic) case, whichever is further landward.

**Block failure of overhanging bluffs and sea caves.** Assessing the factor of safety against block failure for overhanging or notched coastal bluffs, or bluffs undermined by sea caves, is far more difficult than conducting a slope stability analysis against landsliding. This is due to several factors, the most important of which are: 1) uncertainty as to the presence of local heterogeneities or planes of weakness, hidden in the bluff, that commonly control block failures, 2) difficulty in assigning shear strength values to such heterogeneities even if they can be identified, and 3) greater complexity in modeling the stress field within a bluff in terms of heterogeneities or planes of weakness as compared to a modeling a homogenous slope. The current state of the science does not allow for the calculation of a factor of safety against block failure

for such overhanging or notched coastal bluffs, or bluffs undermined by sea caves, and even makes any form of quantitative assessment of the risk of failure extremely difficult. Promise is shown in mathematical models such as that of Belov and others (1999), but translating such process-oriented models into setback methodologies has not yet been attempted.

Accordingly, establishing appropriate setbacks from overhanging or undermined coastal bluffs is problematic at best. An appropriate conservative approach is to project a vertical plane upward from the rear wall of the overhang, notch, or sea cave, and establish this as the minimum setback line. This approach has been adopted by the City of San Diego, and codified in the City's Local Coastal Program. Although it is certainly possible that failure could occur along a line inclined either seaward or landward from the rear wall of the overhang, notch, or sea cave, a vertical plane would seem to be a good default configuration to assume in the absence of more compelling evidence for another configuration. Further, vertical, bluff-parallel fractures—perhaps related to stress-relief at the free face represented by the bluff face are a common feature of otherwise homogenous coastal bluffs. In many cases, such a plane will intersect the sloping bluff face seaward of the bluff edge, and no setback from the bluff edge would be necessary to assure stability from block collapse. In cases where the plane intersects the bluff top seaward of a setback line established for landsliding, as discussed above, no additional setback would be necessary to assure stability from block collapse. In the rather rare case, however, in which the plane intersects the bluff top landward of both the bluff edge and any setback line for landsliding, the line of intersection of the plane and the bluff top would be an appropriate setback line for slope stability considerations.

# Long Term Bluff Retreat

The second aspect to be considered in the establishment of a development setback line from the edge of a coastal bluff is the issue of more gradual, or "grain by grain" erosion. In order to develop appropriate setbacks for bluff top development, we need to predict the position of the bluff edge into the future. In other words, at what distance from the bluff edge will bluff top development be safe from long-term coastal erosion?

The long-term bluff retreat rate can be defined as the average value of bluff retreat as measured over a sufficient time interval that increasing the time interval has negligible effect on the average value (a statistical basis could be applied to the term "negligible," but this is rarely done). This definition implies that the long-term bluff retreat rate is linear, an assumption that certainly is not valid over time scales of more than a few centuries, or in periods of rapid sea-level change such as the late Pleistocene/early Holocene (Curray 1965; Emery and Garrison 1967; Milliman and Emery 1968). There is some overlap between slope stability issues and long-term bluff retreat issues, in that the "grains" may be fairly large rocks, and in that shallow slump-

ing is a common mechanism for gradual bluff retreat. In addition even gradual bluff retreat tends to be highly episodic due to a host of internal and external factors.

The rate at which gradual bluff retreat occurs generally is measured by examining historic data. This is somewhat problematic in that the historic bluff retreat rate may not accurately predict the future bluff retreat rate (Watson 2002). This is a particularly issue in light of the likelihood of an acceleration in the rate of sea level rise as a result of global warming (Intergovernmental Panel on Climate Change 2001) and the resulting likely increase in bluff retreat rate (Bray and Hooke 1997; Watson 2002).

Nevertheless, historic data currently are our best indicators of future erosion at any given site. Such data may include surveys that identify the bluff edge, in which case the criteria used to identify the bluff edge must be the same in the surveys that are compared. Sufficiently detailed surveys are rare, however, and vertical aerial photography is more commonly used to assess changes in bluff position through time. The best data are those compiled photogrammetrically, whereby distortions inherent to aerial photography (due, for example, to tilting of the camera, variations in the distance from the camera to various parts of the photograph, and differences in elevation across the photograph) are corrected (see, for example, Moore 2000). Sometimes such data have been gathered as parts of specific studies of coastal bluff retreat, but more commonly they are collected as part of other work, and must be sought out for coastal erosion studies.

Coastal bluff retreat tends to be temporally episodic due to a variety of external and internal factors. External factors include tides, episodic wave events (spurred by either local or distant storms), episodic rainfall events (Kuhn 2000), El Niño-Southern Oscillation events (Griggs and Johnson 1983; Griggs 1998; Griggs and Brown 1998; Lajoie and Mathieson 1998; Storlazzi and Griggs 2000), major earthquakes (Plant and Griggs 1990; Griggs and Scholar 1997) and long-term climate change on a multidecadal to century scale (Inman and Jenkins 1999). Internal factors include the autocyclicity inherent to many bluff failure mechanisms (Leighton and Associates Inc. 1979; Hampton and Dingler 1998) and bluff response to continued toe erosion (Sunamura 1992).

Despite the episodic nature of coastal bluff retreat, it is necessary to identify the future long-term bluff retreat rate in order to establish appropriate development setbacks. The episodic nature of bluff retreat makes any calculated rate highly dependent on sampling interval. To illustrate the dependence of calculated long-term bluff retreat rates on sampling interval, it is useful to perform a sensitivity analysis from real data. Unfortunately, there are insufficient data to perform a meaningful analysis for any one site in California. Accordingly, a synthetic data set was created as part of this study.

A Synthetic Data Set. Creating and examining a synthetic data set allows for testing the effects of sampling on the determination of long-term bluff retreat rates. The long-term retreat rate is, by definition, known for the synthetic data set. Further, a synthetic data set can be created that is both longer and more complete than any such data set available from nature. The data set considered here (available upon request from the author) was created for a hypothetical 200-year period, assigned the dates 1800-2000. Figure 2 is a graphical representation of the data set, and charts the progressive retreat of the hypothetical bluff edge through that time period. Although the data are fictitious, they roughly correlate with well-known periods of episodic erosion in coastal California, at least for the second half of the data set.

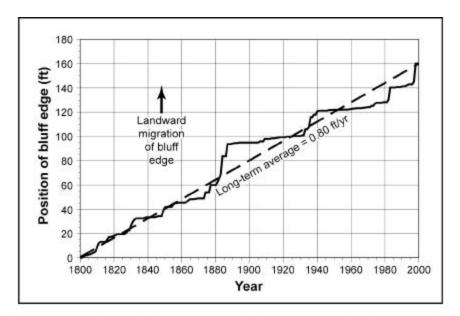


Figure 2. Plot of the position of the top edge of a hypothetical coastal bluff over time. These data represent a synthetic data set that is meant to roughly mimic typical episodic bluff retreat. Although fictitious, the data correlate well with what is know of temporal variations in erosion rate for a typical California bluff experiencing moderate erosion. The data set is far more complete than actual data available at any given site, however, making possible a sensitivity analysis of sampling interval on the calculation of the long-term bluff retreat rate.

**Moving averages.** A standard statistical method to smooth spikes in data is to average the data over a window of some width, while moving that window through the data set. Figure 3 shows the effect of applying this technique to the synthetic data set, using averaging windows of various widths. The first derivative of the curve representing bluff edge position through time (Figure 2) is the "instantaneous" bluff-retreat rate, and varies from 0 to 15 ft/yr for the synthetic data set (Figure 3). As the averaging window increases in width, the maximum retreat rate values decrease and the minimum values increase, effectively smoothing and broadening the "peaks" representing episodic erosion events. Depending on how the window is centered on the point representing the window average, peaks may be offset in time as well. With the widest sampling windows, peaks are essentially eliminated, and the retreat rate calculated approaches the average long-term retreat rate for the entire data set (0.80

ft/yr). Note that it is only when the window width approaches (and exceeds) 50 years in width that the calculated bluff retreat rate approaches the long-term average rate.

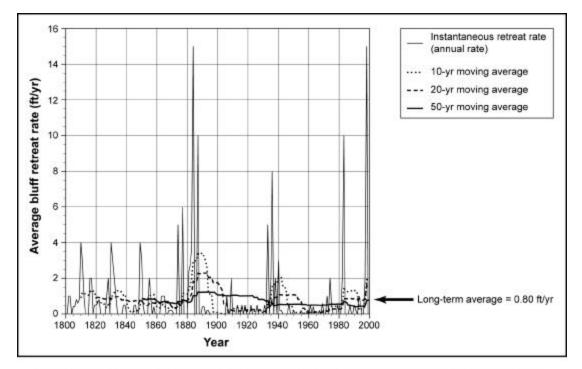


Figure 3. Average annual bluff retreat rate calculated from the synthetic data set using moving averages with various averaging window sizes. Only when data are averaged over ~50 years or more does the calculated annual bluff retreat rate approach the known long-term average for the data set.

**Data gathered at intervals.** Data regarding bluff edge position are almost always gathered at widely spaced intervals, corresponding to the dates of surveys or photographs. This precludes the use of a moving average technique, which depends on continuous data. Figure 4 shows the calculated bluff retreat rates at regularly spaced intervals of 10, 20, and 50 years. A wide range of values for the bluff retreat rate are obtained at the shorter sampling intervals. Although short sampling intervals give the most information on the variability of bluff retreat, the best estimate of the long-term bluff retreat rate is provided by sampling at long time intervals. Even at these long time intervals, if a statistically greater- or lesser-than-average number of "episodic events" are included in the sample, then the bluff retreat rate calculated for that interval will seriously over- or underestimate actual the long-term average bluff retreat rate.

**Principal observations from the synthetic data set.** A few simple generalities can be made from this limited analysis. First, instantaneous bluff retreat rates can exceed the long term average rate by a factor of many times. This is also true for data collected at short (=  $\sim$ 10 years for the synthetic data set) time intervals. Second, data collected at relatively short time intervals give useful information on the episodic nature of bluff retreat, but do not provide accurate estimates of long-term average

bluff retreat rates. Third, the best estimate of long-term average bluff retreat rate is obtained by sampling over long (=  $\sim$ 50 years for the synthetic data set) time intervals. Finally, in order to accurately estimate the long-term bluff retreat rate, a stochastically appropriate number of episodic events must be included in the sampling interval. These observations, as well as similar observations from real data, lead to the general guidelines for estimating the long-term average bluff retreat rate at a site that are presented in Table 2.

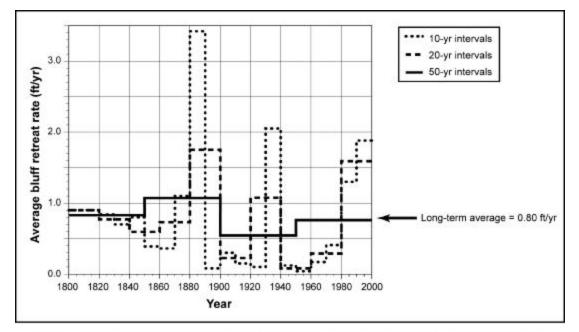


Figure 4. Average annual bluff retreat rate calculated from the synthetic data set using discrete sampling intervals of various sizes. Only when data are sampled at intervals of ~50 years or more does the calculated annual bluff retreat rate approach the known long-term average for the data set.

*Establishing setbacks for long-term bluff retreat.* Once an historic long-term bluff retreat rate has been estimated, establishing a setback for long-term bluff retreat rate is a simple matter of multiplying that rate, *B*, by the design life of the development, *t*. This is equivalent to predicting the position of the coastal bluff edge at the end of the design life of the structure (Figure 5).

Although this is the usual method of establishing setbacks for long-term bluff retreat in California, inherent assumptions and difficulties must be born in mind. Foremost among these is the necessity of defining the design life of the development. Because the landward retreat of an unarmored shoreline is inevitable and ongoing during a period of relative sea level rise, it is impossible to assure the safety of development from coastal erosion unless a time frame is assigned at the onset. But assigning a design life is difficult, and there is nothing in land use law that requires the abandonment of development at the end of its assigned design life. Other problems associated with this type of analysis revolve around its inherently historic approach. There is no *a priori* reason to believe that bluff retreat rates are, or will continue to be, linear. This is especially relevant in light of expected acceleration of the historic rate of sea level rise as a result of global warming (Intergovernmental Panel on Climate Change 2001). Further, there is good evidence that erosion rates can be highly variable through time (Jones and Rogers 2002). For all of these reasons it is important to adopt a conservative approach to estimating long-term bluff retreat rates.

# Table 2. Guidelines for establishing long-term bluff retreat rates

- Determine bluff edge positions at as many times as possible, but covering a minimum of about 50 years and extending to the present. Common data sets include vertical aerial photographs, surveys that identify the bluff edge, and detailed topographic maps. These sources must be of sufficient scale or precision to locate accurately the position of the bluff edge to within a few feet.
- 2) If aerial photographs are used, the best results are obtained through photogrammetric methods, whereby distortions inherent to aerial photography are corrected (orthorectified). Even if photogrammetric methods are not used, the scale of the photographs must be carefully determined by comparison of the image size of known features to their actual size.
- 3) When comparing bluff edge positions on aerial photographs or unanchored surveys, a "shoreline reference feature" must be identified that has been static through time and is identifiable in each data set. Bluff positions throughout the area of reference can be measured relative to this feature. Common shoreline reference features are road centerlines, structures, large rock outcrops, or trees.
- 4) When comparing bluff edge positions on surveys, it is critical that the same criteria for the identification of the bluff edge was used in each survey. The Coastal Act definition of a bluff edge can be found in California Code of Regulations, Title 14, § 13577 (h) (2).
- 5) Although the short-term erosion rate for each time interval between data points provides valuable information regarding the nature of bluff retreat at the site, the long-term erosion rate should be determined from the extreme end-points of the time series examined. This time series should exceed 50 years in length, and should include both relatively quiet periods, such as the 1950's-1960's; and the more erosive subsequent time periods (especially the 1982-1983 and 1997-1998 El Niño winters).
- 6) In larger study areas, the bluff retreat rate should be determined at intervals along the bluff edge, paying special attention to potential differences in retreat rate between headlands and coves, and amongst areas underlain by differing geologic materials.

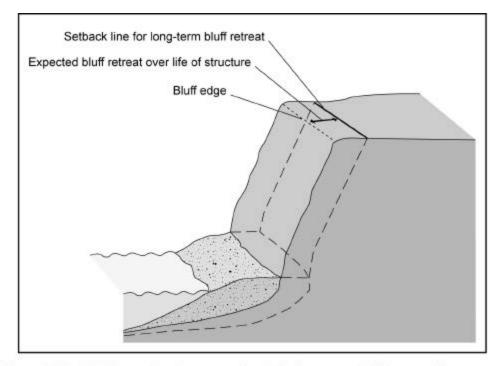


Figure 5. Establishing a development setback for long term bluff retreat. The expected bluff position at the end of the development's useful life is found by multiplying the average annual bluff retreat rate by the design life of the development; this line is taken to represent the minimum setback for long-term bluff retreat.

# Uncertainty

There is a great deal of uncertainty in many parts of the analysis discussed above. The deterministic approach outlined here does not deal well with such uncertainty. Various methods have been used to build in some margin for error in establishing safe building setbacks. One approach, commonly used by geologists working in northern California, is to multiply the long-term bluff retreat rate by a factor of safety (used in a different sense than for slope stability), generally ranging from 1.5 to 4.0. More commonly, a simple "buffer" is added to the setback generated by multiplying the long-term bluff retreat rate by the design life of the structure. This buffer, generally on the order of ten feet, serves several functions: 1) it allows for uncertainty in all aspects of the analysis; 2) it allows for any future increase in bluff retreat rate due, for example, to an increase in the rate of sea level rise (Bray and Hooke 1997; Watson 2002); 3) it assures that at the end of the design life of the structure the foundations are not actually being undermined (if that were to be the case the structure would actually be imperiled well before the end of its design life); and 4) it allows access so that remedial measures, such as relocation of the structure, can be taken as erosion approaches the foundations. If a slope stability setback is required (*i.e.*, if the bluff does not meet minimum slope stability standards), that setback can do double duty as this buffer.

# Summary: Defining the Total Setbacks for Bluff-Top Development

To define the total development setback, one must combine the two aspects of the setback considered above: the setback to assure safety from landsliding or block failure, and the setback for long-term bluff retreat. The resulting setback assures that minimal slope stability standards are maintained for the design life of the structure.

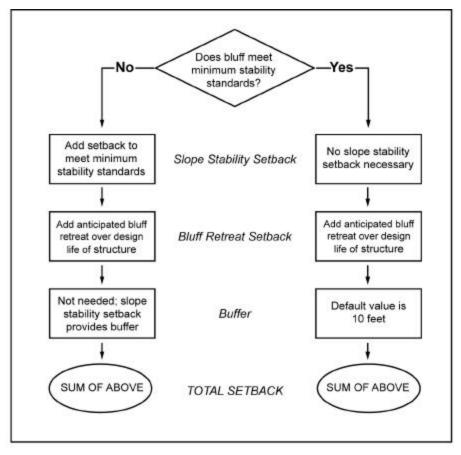


Figure 6. Flowchart for establishing bluff edge setback for development, taking into account stability of the bluff, long-term bluff retreat, and uncertainty in the analysis.

A methodology for combining these setbacks is outlined in Figure 6. First, it must be determined whether the coastal bluff meets minimum slope stability standards. Normally, this will be a factor of safety of 1.5 (static) or 1.1 (pseudostatic). If the answer to this question is "yes," then no setback is necessary to assure slope stability. If the answer is "no," then it is necessary to determine the position on the bluff top where the minimum slope stability standards are attained. This position, as measured relative to the bluff edge, is the setback necessary for slope stability determined as described above. In the case of block failure of an overhanging bluff or collapse of a sea cave, the setback necessary to assure stability from this type of collapse is equivalent to the slope stability setback. Although the current state of the science makes it impossible to quantitatively assess stability relative to this type of failure, a conservative, yet realistic, setback line is the projection of a vertical plane from the rear wall of the overhang or sea cave on the bluff top. If the plane does not intersect the bluff top (*i.e.*, intersects the inclined bluff face seaward of the bluff edge), then no setback for this type of collapse is necessary.

The next step is to determine the expected bluff retreat over the design life of the structure, as described above. This setback is added to the slope stability setback, if any.

Finally, a buffer, generally a minimum of 10 feet, should be added to address uncertainty in the analysis, to allow for any future increase in the long-term bluff retreat rate, to assures that the foundation elements aren't actually undermined at the end of the design life of the development, and to allow access for remedial measures. A buffer is not necessary if the slope stability setback equals or exceeds about ten feet, as it can do "double duty" as both a setback to assure slope stability and a buffer for the purposes listed above.

The total setback is meant to assure that minimum slope stability standards are maintained for the design life of the development. Inherent in this analysis is the assumption that factors affecting slope stability (steepness and shape of the slope, ground water conditions, geometry of rock types exposed in the bluff) will remain constant through the design life of the development, that the future bluff-retreat rate will be linear and of comparable magnitude to the historic rate, and that the nature of erosion processes at the site will remain unchanged. All of these assumptions are potentially flawed, but in the absence of convincing evidence to the contrary, are a means of establishing reasonable development setbacks.

# **Towards Probabilistic Coastal Erosion Hazard Assessment**

The deterministic approach presented above is based on established geologic and engineering principals, and similar approaches have been used to establish development setbacks from slope edges throughout the world for some time. However, the approach suffers from its limited ability to consider uncertainties in the analysis. Probabilistic approaches, on the other hand, inherently consider analytical uncertainties, and allow for a better definition of risk. This type of risk assessment has been routine for decades in the field of hydrology, where design basis and land use priorities are based on the magnitude of the "100-year flood," for example. Probabilistic coastal hazard assessment similarly can be used to quantify the likelihood that the bluff edge will erode to any particular point on a bluff top in a given time. Then, by establishing an acceptable level of risk (for example, a probability of <5% that the bluff edge will reach a certain point over the design life of the development) a setback line can be established that inherently includes uncertainties in the analysis. Just as the seismological community has moved away from deterministic methods towards probabilistic ones, such an approach allows for better consideration of the uncertainties in estimating future coastal erosion.

Probabilistic coastal hazard assessment is in its infancy, and no standardized methods have won acceptance—or even much discussion. The failure of coastal bluffs along Lake Michigan through landsliding has been assessed probabilistically by Chapman and others (2002), through the use of probabilistic slope stability analyses. Lee and others (2001) applied a variety of probabilistic methods to questions of coastal bluff retreat in England. Methods that they evaluated include the simulation of recession of episodically eroding cliffs through Monte Carlo techniques, the use of historical records and statistical experiments to model the behavior of cliffs affected by episodic landslide events, event-tree approaches, and the evaluation of the likelihood of the reactivation of ancient landslides. All of these techniques show promise, but the authors restricted themselves to specific cases. What is needed is the development of probabilistic methods that will work in more general cases, and combine both slope stability and long-term bluff retreat considerations. One way to approach this problem is to consider separately the two aspects of defining a development setback as outlined above.

Probabilistic slope stability analyses already are routine (Mostyn and Li 1993; Yang et al. 1993). In addition to quantifying the probability of slope failure (something not done in a deterministic slope stability analysis, which only establishes whether or not failure will occur), probabilistic slope stability analysis allows for consideration of variability or uncertainty in soil or rock strength parameters (Lumb 1970). Uncertainties in these input parameters are quantified by the standard deviation of each parameter. Then, using Monte Carlo techniques, a probability distribution for the factor of safety associated with any given failure plane is produced. From this, the probability of failure along the chosen potential failure plane can be calculated. The probability of failure is the probability that the factor of safety will be less than 1.0, and can be calculated for any given potential failure surface. By performing such analyses on a variety of potential failure surfaces intersecting different portions of the bluff top, a probability could be assigned to any position on the bluff top quantifying the likelihood that a failure will occur landward of that point.

Although not routine, several possibilities present themselves for developing probabilistic models for gradual, episodic, bluff retreat. Perhaps the simplest method of quantifying uncertainty is the application of a confidence interval to the estimate of the long-term average bluff retreat rate. Each time interval examined in estimating this rate is one sample of the mean value. For normally distributed data (or data that can be transformed to a normal distribution by, for example, a log transform), the sample standard deviation is a traditional estimate of uncertainty. There is a ~68.26% probability that the true mean value will lie within  $\pm 1$  standard deviation of the sample mean. Different probabilities apply to different multiples of the standard deviation. Thus, uncertainties in the product ( $B \ge t$ ), above, can be quantified and contoured on the bluff top. For populations that cannot be shown to be normally distributed (likely the case with the small sample sizes available for bluff retreat rates), a better estimate of uncertainty may be a confidence interval based on Student's t distribution, or on no nparametric statistics. A second approach to probabilistic assessment of coastal bluff recession is to treat annual bluff retreat in a manner analogous to river floods. Thus, the recurrence interval of a particular amount of annual bluff retreat can be calculated by the formula

$$R = \frac{N+1}{M}$$

where R is the recurrence interval, N is the number of years of record, and M is the rank of the annual bluff retreat in the total data set. For the synthetic data set considered above, there are many duplicate values due to the limited precision with which bluff retreat data are generally reported. Eliminating duplicates, and ranking the annual bluff retreat rates, recurrence intervals can be calculated. These data can be graphed in order to arrive at the expected amount of bluff retreat for any particular recurrence interval (Figure 7). The inverse of the recurrence interval is the annual probability that a given amount of bluff retreat will be exceeded. Such data may be especially valuable in assessing the risk of occurrence of an episodic event sufficient to threaten an existing structure.

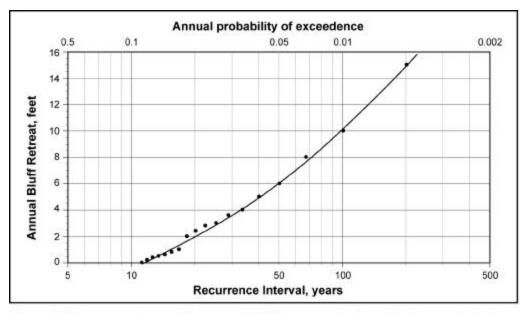


Figure 7. Recurrence interval for annual bluff retreat, calculated for the synthetic data set. The recurrence interval, calculated in a manner analogous to flood recurrence interval, gives the average time between years with a given amount of bluff retreat. The inverse of the recurrence interval is the statistical probability that a given amount of bluff retreat will occur (or be exceeded) in any given year.

The total risk to bluff-top development, which includes both long-term bluff retreat and slope failure, can be calculated by multiplying the probability of slope failure at a given position by the probability that bluff retreat will reach that point by a given time. The geotechnical and planning communities will need to establish what is an acceptable probability, or risk, that the bluff will reach a given point in order to develop setback criteria. Once that probability is established, the setback line can be defined as the locus of points on the bluff top at that probability.

A prime difficulty in applying probabilistic methods to assessing coastal erosion risk will be the difficulty in acquiring sufficiently rich data sets with which to work. More effort is needed at acquiring long, precise data sets on coastal erosion in a variety of geologic conditions throughout the state.

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